SVKM's
D. J. Sanghvi College of Engineering

Academic Year: 2022

Duration: 3 hours

Program: B.Tech in Computer Science and Engineering (Data Science)
Date: 19.01.2023
Time: 09:00 am to 12:00 pm
Subject: Mathematics for Intelligent Systems (Semester III)

Marks: 75

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.
(1) This question paper contains two pages.
(2) All Questions are Compulsory.
(3) All questions carry equal marks.
(4) Answer to each new question is to be started on a fresh page.
(5) Figures in the brackets on the right indicate full marks.
(6) Assume suitable data wherever required but justify it.
(7) Draw the neat, labelled diagrams, wherever necessary.

Question No.		Max. Marks
Q1 (a)	$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$	
	i) Is A derogatory?	[04]
	ii) Find the eigenvalues of $A^3 - 3A + 9I$	[03]
	OR	
	$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$	
	i) Use Cayley-Hamilton Theorem to find $A^4 - 3A^2 + 5A - I$	[04]
	ii) Find A^{100}	[03]
Q1 (b)	Solve the following LPP using Big M method Minimize $z = 600x_1 + 500x_2$ Subject to $2x_1 + x_2 \geq 80$ $x_1 + 2x_2 \geq 60$ $x_1, x_2 \geq 0$	[08]

Q2 (a)	Derive the moment generating function of Poisson Distribution and hence find the mean and variance **OR** Solve the following LPP using Simplex method Maximize $z = 40x_1 + 30x_2$ Subject to $x_1 + x_2 \leq 12$ $2x_1 + x_2 \leq 16$ $x_1, x_2 \geq 0$	[07] [07]
Q2 (b)	The average number of acres burned by forest and range fires in a large county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. i) What is the probability that between 2,500 and 4,200 acres will be burned in any given year? ii) What is the probability that less than 3,000 acres will be burned in a year? iii) What is the probability that more than 5,000 acres will be burned in a year? iv) What number of burnt acres corresponds to the 38th percentile?	 [02] [02] [02] [02]
Q3 (a)	Find the first four raw moments and the first four central moments for the following continuous random variable with the PDF $f(x) = 12(x^2 - x^3); 0 < x < 1$ **OR** i) Use Lagrange's multiplier method to optimize $z = x_1^2 - 12x_1 + 2x_2^2 - 12x_2 + 60$ subject to $x_1 + x_2 = 9$ $x_1, x_2 \geq 0$ ii) Find the dual of the primal Minimize $z = 2x_1 + 3x_2 + 5x_3$ Subject to $x_1 - 2x_2 + 4x_3 \geq 6$ $-x_1 + x_2 + x_3 \leq 7$ $x_1, x_2 \geq 0, x_3$ is unrestricted	[07] [04] [03]
Q3 (b)	i) Find the basis and dimension of $W^\perp$ where $W = \{(0, 1, -2, 1), (5, 2, 4, 1), (5, 3, 2, 2)\}$ ii) If $S = \{(1, 2, 0), (0, 1, 2), (1, 1, 1)\}$ and $v = (-1, 2, 3)$ find the coordinate vector relative to the basis S i.e., $[v]_S$	[04] [04]

Q4 (a)

i) Prove that the vectors are linearly independent [03]

$$v_1 = (1,3,4), v_2 = (3,-5,2), v_3 = (2,2,4)$$

ii) Find the basis and dimension of the solution space of the system of equations: [04]

$$x_1 + x_2 - x_3 + x_4 = 0$$
$$x_1 - x_2 + 2x_3 - x_4 = 0$$
$$2x_1 + x_3 = 0$$

OR

i) Find the basis and dimension of the null space, column space and row space of $A = \begin{bmatrix} 1 & -3 & 2 & 3 & -1 \\ -3 & 9 & -1 & 1 & -7 \\ 2 & -6 & 5 & 8 & -4 \end{bmatrix}$ [04]

ii) If $T(2,3) = (4,5)$ and $T(1,-1) = (2,3)$ find A and $T(2,0)$ [03]

Q4 (b)

Prove that A is diagonalizable and hence find the diagonalizing matrix M and diagonal matrix D [08]

$$A = \begin{bmatrix} 0 & -6 & -4 \\ 5 & -11 & -6 \\ -6 & 9 & 4 \end{bmatrix}$$

Q5 (a)

Solve the following NLPP using Karush Kuhn Tucker Conditions [07]

Minimize $z = 7x_1^2 + 5x_2^2 - 6x_1$

Subject to $x_1 + 2x_2 \leq 10$

$x_1 + 3x_2 \leq 9$

$x_1, x_2 \geq 0$

OR

Find k and the covariance between X and Y given the following joint PMF [07]

X / Y	1	2	3
1	$\frac{1}{k}$	$\frac{2}{k}$	$\frac{3}{k}$
3	$\frac{3}{k}$	$\frac{6}{k}$	$\frac{9}{k}$
5	$\frac{5}{k}$	$\frac{10}{k}$	$\frac{15}{k}$

Q5 (b)

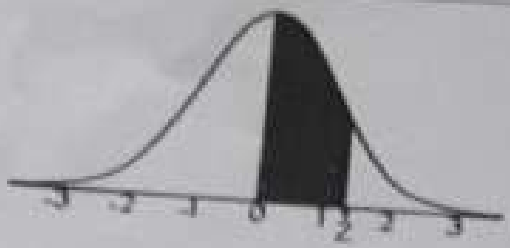
i) Find the directional derivative of f at $(0,-1,2)$ in the direction of $(3,-3,3)$ [03]

$$f(x_1,x_2,x_3) = x_1^2 x_2^4 - \frac{6x_3}{x_2}$$

ii) Determine if the following function is convex, concave, or neither. [02]

$$f(x_1,x_2) = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$$

	iii) On an average, the waiting time to receive a green card can last 6 years for an Indian. The waiting time to receive a green card is exponentially distributed. Eighty percent of Indians wait at most how long to receive a green card?	[03]



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998